21cm bispectrum as method to measure cosmic dawn and EoR

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based on
Introduction

- $z > 30$: Dark age: There are no luminous objects.
- $15 < z < 30$: Cosmic dawn: After star formation, astrophysical effects become effective.
- $7 < z < 15$: Epoch of Reionization (EoR): Hydrogen is ionized by UV photon emitted from stars, galaxies.
21cm line

We would like to observe cosmic dawn (CD) and EoR directly.

Many hydrogen atoms exist in the IGM during CD and EoR.

cosmological redshifted 21cm signal!

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Furlanetto & Briggs 2004
21cm line

21cm line radiation: Neutral hydrogen emits the radiation due to the hyperfine structure.

\[ \Delta E = 5.9 \times 10^{-6} \text{eV} \]

\[ \lambda = 21 \text{cm} \]
\[ \nu = 1.42 \text{GHz} \]

\[ z=8: \lambda = (1+8) \times 0.21 = 1.89 \text{m}, \quad \nu = 159 \text{ MHz} \]

\[ z=27: \lambda = (1+27) \times 0.21 = 5.8 \text{m}, \quad \nu = 51 \text{ MHz} \]
Spin temperature

- **Spin temperature**: spin temperature is defined by the ratio of the number of electrons in the triplet and singlet states.

\[
\frac{n_1}{n_0} = 3 \exp \left( - \frac{h\nu_{21}}{kT_S} \right) \quad \Rightarrow \quad T_S^{-1} = \frac{T_{\text{CMB}}^{-1} + (x_\alpha + x_K)T_K^{-1}}{1 + x_\alpha + x_K}
\]

Spin temperature is determined by collision of H atom

- **with CMB photons**
- **with Ly-\(\alpha\) photons**
- **with other H atoms**
WF effect

Wouthuysen-Field effect

1S spin state changes via 2P states by absorption of Ly-alpha photon

Total angular momentum
\[ \Delta F = 0, 1 \] is permitted
\[ F = 0 \rightarrow 0 \] is not allowed

Transitions represented by solid lines change spin state. However, dashed line does not change spin state.
Thermal history

Mesinger et al 2010

\[ T_{S}^{-1} = \frac{T_{CMB}^{-1} + x_\alpha T_{\alpha}^{-1} + x_K T_{K}^{-1}}{1 + x_\alpha + x_K} \]

- **Blue**: CMB temperature \( \propto (1 + z) \)
- **Green**: kinetic temperature
- **Red**: spin temperature
Thermal history

Mesinger et al 2010

\[ T_S^{-1} = \frac{T_{CMB}^{-1} + x_\alpha T_\alpha^{-1} + x_K T_K^{-1}}{1 + x_\alpha + x_K} \]

X-ray heating

collision coupling

collision decoupling

WF effect

Spin temperature couples kinetic temperature via Ly-α photons
Brightness temperature

We actually observe brightness temperature not spin temperature

\[ \delta T_b(\nu) = \frac{T_S - T_\gamma}{1 + z}(1 - e^{-\tau_\nu_0}) \]

\[ \sim 27x_H(1 + \delta_m)\left(\frac{H}{dv_r/dr + H}\right)\left(1 - \frac{T_\gamma}{T_S}\right)\left(1 + z\frac{0.15}{10\Omega_m h^2}\right)^{1/2}\left(\frac{\Omega_b h^2}{0.023}\right)[\text{mK}]. \]

\( x_H \): neutral hydrogen fraction

\( \delta_m \): matter density fluctuation

\( dv_r/dr \): velocity gradient of IGM
21cm power spectrum

\[ \langle \delta T_b(k) \delta T_b(k') \rangle = (2\pi)^3 \delta(k + k') P_{21} \]

We often use 21cm power spectrum to extract astrophysical information.

We can see three peaks in the 21cm power spectrum.

21cm power spectrum teaches us that the epoch of WF effect, X-ray heating, EoR.
Brightness temperature distribution

(ex)
spacial distribution of brightness temperature @ z=10

histogram of $\delta T_b$

Brightness temperature follows non-gaussian distribution.

estimate non-gaussianity by bispectrum
Formalism

Definition

\[ \langle \delta T_b(k_1) \delta T_b(k_2) \delta T_b(k_3) \rangle = \delta(k_1 + k_2 + k_3) B(k_1, k_2, k_3) \]

- **Equilateral type**

  \[ |k_1| = |k_2| = |k_3| = k \]

- **Squeezed type**

  \[ |k_1| = |k_2| = k, |k_3| = k_c \quad (k > k_c) \]

- **Folded type**

  \[ k_3 = 2k_1 = 2k_2 \]
Redshift dependence

Squeezed type 21cm bispectrum has three peaks. This type is different from other types of bispectrum.

equilateral
\[ k_1 = k_2 = k_3 = 1.0 \text{Mpc}^{-1} \]

folded
\[ k_1 = k_2 = 1.0 \text{Mpc}^{-1}, k_3 = 2.0 \text{Mpc}^{-1} \]

squeezed
\[ k_1 = k_2 = 1.0 \text{Mpc}^{-1}, k_3 = 0.1 \text{Mpc}^{-1} \]
Decomposition

decompose brightness temperature into each fluctuation

\[
\delta T_b(x) = \overline{\delta T_b}(1 + \delta_{xH}(x))(1 + \delta_m(x))(1 + \delta_\eta(x)),
\]

neutral fraction  matter fluctuation  spin temperature

\[
\eta = 1 - T_\gamma/T_S
\]

Bispectrum

\[
B_{\delta T_b} = (\overline{\delta T_b})^3[B_{\delta m\delta m\delta m} + B_{\delta_{xH}\delta_{xH}\delta_{xH}} + B_{\delta_\eta\delta_\eta\delta_\eta} + \text{(cross correlation terms)} + \text{(higher order terms)}].
\]
21cm Bispectrum contour

○ Fix one side of triangle. Others are free parameter.
○ We calculate the bispectrum for each configuration of triangle.
○ Symmetry for dashed line.
○ If triangular condition can not be satisfied, we can not calculate bispectrum (white space)
21cm Bispectrum contour

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21cm Bispectrum contour

\[ k_c = 1.0 \text{Mpc}^{-1} \]

\[ k_c = 0.4 \text{Mpc}^{-1} \]

○ unnormalized by \( k \)

○ We can see what component and configuration are characteristic.

(ex.) At \( z=10 \), folded type & squeezed type of bispectrum are strong. Dominant component comes from neutral hydrogen fraction.
21cm Bispectrum contour

\[ k_c = 1.0 \text{Mpc}^{-1} \]

At large scales, the effect of matter fluctuation is small compared with smaller scales.

We may extract the information of each component by means of the dependence on configuration.
Summary

- 21cm bispectrum is one of the methods to estimate non-gaussianity of 21cm signal.

- We can see correlation between large and small scales with squeezed type of 21cm bispectrum.

- The component of 21cm bispectrum depends on the configuration of triangle.
Parameter constraint

(on-going work)
Fisher analysis

We often use Fisher analysis to forecast expected errors for observations.

\[ F_{ij} \equiv -\left\langle \frac{\partial^2 \ln L}{\partial \theta_i \partial \theta_j} \right\rangle = \sum_{k,z} \frac{1}{\varepsilon^2(k, z)} \frac{\partial \Delta^2(k, z)}{\partial \theta_i} \frac{\partial \Delta^2(k, z)}{\partial \theta_j}, \]

\( \varepsilon(k, z) \) : Error on \( \Delta^2(k, z) \)

\( \bar{\theta} \) : Parameter vector

Given certain fiducial model, we can estimate 1 sigma error on parameter by \( (F^{-1})_{ii} \)
We focus on following EoR model parameters and we assume the LOFAR and MWA(256T) observations at $z=7,8,9$.

$\zeta$ : the ionizing efficiency. (fiducial value = 15.0)

This parameter characterizes the number of photon escaping from high $z$ galaxy.

$$\zeta = \frac{f_{esc} f_* N_\gamma}{1 + n_{rec}}$$

$T_{vir}$ : the minimum Virial temperature of halos producing ionizing photons (fiducial value = $1.0 \times 10^4$ K)
Fisher analysis for BS & PS

\[ \sigma_{T_{\text{vir}}}(z = 7, \text{MWA}) = 6.17 \times 10^3 \]
\[ \sigma_{\zeta}(z = 7, \text{MWA}) = 3.71 \]

\[ \sigma_{T_{\text{vir}}}(z = 7, \text{LOFAR}) = 869 \]
\[ \sigma_{\zeta}(z = 7, \text{LOFAR}) = 0.605 \]
Summary

- There are power spectrum, one-point statistics and bispectrum as method to probe 21cm signal statistically.

- We can see correlation between large and small scales with squeezed type of 21cm bispectrum.

- The component of 21cm bispectrum depends on the configuration of triangle.

- We perform the Fisher analysis for EoR model parameters assumed LOFAR and MWA. We can obtain tighter constraint from bispectrum than power spectrum.

- LOFAR can put tighter constraint on EoR model parameters than MWA.

- We have to consider sample variance and foreground.
Back up
Constraints

The observation of the CMB polarization

→the optical depth of Thomson scattering

\[ \tau_e \propto \int_{z_r}^{0} n_e(z) \frac{dt}{dz} dz \]

We can estimate the number density of free electrons from the optical depth.

\[ \tau = 0.066 \pm 0.012 \quad \zeta_{re} = 10.5 \pm 1.1 \quad \text{(Planck 2015)} \]
Constraints

high-z QSO absorption

→ constraint on the epoch where the EoR finishes.

Gun-Peterson test

If the neutral hydrogen exists, it absorbs the Ly-alpha photons.

No emission line!

We can know the epoch where EoR finished via the QSO spectrum.
Constraints

At higher redshift over than \( z \approx 7 \)

We can use Ly\( \alpha \) emitter galaxies (LAE) as probe of EoR.

Luminosity function@\( z = 7.3 \)

\[ x_{\text{HI}} = 0.3 - 0.8 \]

(Konno et al 2014)
Imaging maps of the 21cm signal

$z=30$
$x_H=0.999$
$\text{ave}=-8.4\text{mK}$

$z=20$
$x_H=0.996$
$\text{ave}=-171\text{mK}$

$z=15$
$x_H=0.951$
$\text{ave}=-43\text{mK}$

$z=10$
$x_H=0.374$
$\text{ave}=6.9\text{mK}$
Introduction

Thermal history

Mesinger et al 2010

\[
T_S^{-1} = \frac{T_{\text{CMB}}^{-1} + x_\alpha T_\alpha^{-1} + x_K T_K^{-1}}{1 + x_\alpha + x_K}
\]

Determined by both astrophysics and cosmology

Determined by cosmology
Evolution of 21cm power spectrum
Method

- We use 21cmFAST (Mesinger et al 2010) to calculate brightness temperature field (200Mpc$^3$, 300$^3$ grid)

- Zel’dovich approximation for calculation of matter density field

  +

- Use analytic model for evolution of ionized field and heating process

- Free parameter → ionizing efficiency, the number of photon emitted from stars, the number of X-ray photon
Decomposition of 21cm power spectrum

Decomposition of 21cm power spectrum into each component

brightness temperature → (average) + (fluctuation)

\[ \delta T_b = (\overline{\delta T_b})(1 + \delta_m)(1 + \delta_{xH})(1 + \delta_\eta) \quad \eta = 1 - T_\gamma/T_S \]

calculate each power spectrum

\[ \langle \delta_m(k)\delta_m(k') \rangle = (2\pi)^3 \delta(k+k')P_m(k). \]

\[ \langle \delta_H(k)\delta_H(k') \rangle = (2\pi)^3 \delta(k+k')P_{xH}(k). \]

\[ \langle \delta_\eta(k)\delta_\eta(k') \rangle = (2\pi)^3 \delta(k+k')P_\eta(k). \]
Decomposed 21cm power spectrum

- Three peaks are appeared.
- The component $x$ produces a peak at EoR.
- The component $\eta$ produces two peaks.
- The matter component also contributes to power spectrum.
Decomposed 21cm power spectrum

At EoR, the fluctuation of $X_{\{H\}}$ is dominant. $\eta$ fluctuation is effective before EoR

$\rightarrow$ WF effect, X-ray heating

We try to understand spin temperature physically.
One-point statistics
Evolution of $\eta$

We focus on the region where the matter density is large. Luminous objects easily form and drive astrophysical effects more effective than other low density regions. Around the objects, WF effect is caused by UV photons first and X-ray heating is effective later.
PDF (probability distribution function) of $\eta$

- Tail corresponds to the region where luminous objects form.
- Higher redshift $\rightarrow$ tail toward low eta. (WF effect)
- Lower redshift $\rightarrow$ tail toward high eta. (X-ray heating)
- Transition of tail $\rightarrow$ nearly gaussian distribution and small width. (X-ray heating begins effective.)
Variance and skewness

○ variance and skewness

\[ \sigma^2 = \frac{1}{N} \sum_{i=1}^{N} [X - \bar{X}]^2 \]

\[ \gamma = \frac{1}{N \sigma^3} \sum_{i=1}^{N} [X - \bar{X}]^3 \]

left side tail \(\rightarrow\) negative
right side tail \(\rightarrow\) positive

○ variance \(\rightarrow\) reflect length of the tail (The period that WF effect is effective)
○ skewness \(\rightarrow\) change its sign by X-ray heating
Variance and skewness of brightness temperature

We actually observe brightness temperature.

variance

\[ \sigma_{\delta T} = (\bar{T})^2 \left[ \sigma_{\delta m} + \sigma_{\delta \eta} + \sigma_{\delta x_H} + \langle \delta_m \delta_\eta \rangle + \langle \delta_m \delta_{x_H} \rangle + \langle \delta_\eta \delta_{x_H} \rangle + O(\delta^3) \right]. \]

skewness

\[ \gamma_{\delta T} = (\bar{T})^3 \left[ \gamma_{\delta m} + \gamma_{\delta \eta} + \gamma_{\delta x_H} + \langle \delta_m \delta_\eta \delta_{x_H} \rangle + 3(\langle \delta_m^2 \delta_\eta \rangle + \langle \delta_m \delta_{x_H} \rangle + \langle \delta_\eta^2 \delta_{x_H} \rangle + \langle \delta_m \delta_\eta^2 \rangle + \langle \delta_m \delta_{x_H}^2 \rangle + \langle \delta_\eta \delta_{x_H}^2 \rangle) + O(\delta^4) \right]. \]

Plot the auto correlation term of variance & skewness
Variance and skewness of brightness temperature

- The change of sign at skewness is different between $\eta$ and $\delta T$ due to the matter fluctuation. But basic physical interpretation is same.
- Variance and skewness become good indicator for WF effect and X-ray heating.
Summary

- We decompose 21cm power spectrum into each component and confirm that the $\eta$ is effective before EoR.

- Variance and skewness become good indicator for WF effect and X-ray heating.

- We need to evaluate higher order term $\rightarrow$ bispectrum
Constraints

high-z QSO absorption
\[ \rightarrow \text{constraint on the number of neutral hydrogen.} \]

(Fan et al 2006)

However, absorption line is easily saturated where ratio of neutral hydrogen is less than \( \sim 0.001 \) due to its large optical depth.

\[ \tau_{GP}(z) = 4.9 \times 10^5 \left( \frac{\Omega_m h^2}{0.13} \right)^{-1/2} \left( \frac{\Omega_b h^2}{0.02} \right) \left( \frac{1 + z}{7} \right)^{3/2} \left( \frac{n_{\text{HI}}}{n_H} \right) \]

\[ f_{\text{HI}} < 0.001(z = 6.1) \]

We can not use this method \( z > \sim 6 \)
Scale dependence

This difference comes from dominant component.

At $z=20$ matter fluctuation else spin temperature fluctuation or neutral hydrogen fluctuation

- At $z=10, 15, 27$
  nearly scale invariant

- At $z=20$
  Increase with wave number
Fisher analysis for PS

\[ \sigma_{T_{\text{vir}}}(z = 7, \text{MWA}) = 1.61 \times 10^6 \]
\[ \sigma_{\zeta}(z = 7, \text{MWA}) = 813 \]
\[ \sigma_{T_{\text{vir}}}(z = 7, \text{LOFAR}) = 3.39 \times 10^5 \]
\[ \sigma_{\zeta}(z = 7, \text{LOFAR}) = 171 \]
Fisher analysis for BS

MWA

LOFAR

\[ \sigma_{T_{\text{vir}}}(z = 8, \text{MWA}) = 2.37 \times 10^3 \]

\[ \sigma_{T_{\text{vir}}}(z = 8, \text{LOFAR}) = 70.9 \]

\[ \sigma_{\zeta}(z = 8, \text{MWA}) = 1.28 \]

\[ \sigma_{\zeta}(z = 8, \text{LOFAR}) = 0.057 \]
Various X-ray models

change the number of X-ray photon
($\zeta_X=10^{57}, 10^{56}, 10^{55}/M_{\text{sun}}$)

Peak of variance and skewness is shifted $\rightarrow$ distinguish models.
**Noise estimation**

**Power spectrum** (McQuinn et al. 2006)

(u,v,η)-spaceでvisibilityとintensityを定義。

single base lineでのnoise intensityに対するnoise power spectrumを計算。

multiple base lineからのnoiseへの寄与を計算。

**Bispectrum** (Yoshiura et al. 2015)

Thermal noiseはgaussianを仮定するため、noise bispectrumのアンサンブル平均は0。しかし、bispectrumのvarianceはnon zeroなため、noiseとして効いてくる。

Bispectrumのvarianceを定義して計算。

power spectrumのnoiseと同様にmultiple base lineからのnoiseへの寄与を計算。
Signal vs Noise for PS

Power spectrum と 感度曲線 @ $z=7,8,9$

MWA, LOFAR, HERA, SKA を想定
Signal vs Noise for BS

Bispectrumと感度曲線 @z=7,8,9

MWA,LOFAR,HERA,SKAを想定
○EoRパラメータを変化させた時のbispectrumの変化@k=0.1Mpc^{-1}。
○イオン化効率が大きい→イオン化の時期が早くなるため、peakはhigh-zにシフト。
○ビリアル温度が大きい→天体形成が遅れるためイオン化も遅れ、peakはlow-zにシフト。
The ratio of differential coefficient

\[ r = \left( \frac{\partial \Delta B}{\partial p_i} \right)^2 / \left( \frac{\partial \Delta P}{\partial p_i} \right)^2 \]

○各\( z\)での微分係数の比。
○bispectrumの変化率の方がpower spectrumの変化率よりも大きい。
○\( z=7 \)の時の変化率が\( z=7, 8, 9 \)の中で一番小さい。
bispectrumとpower spectrumでのエラーの比。

bispectrumは変化率も大きいが、Noiseも大きい。しかし、正味としてはbispectrumの方がpower spectrumにFisher matrixの値が大きい。